Extensive-form games: applications

- Stackelberg model
- Spence-Dixit model
- Rubinstein bargaining model

Stackelberg model

- Consider a Stackelberg duopoly game with symmetric technologies.
- There are two inputs K (capital) and L (labor).
- Firms have the same Leontief production functions:

$$q_i = \min\{L_i, K_i\}$$

- The cost of L is *w* and the cost of K is *r*, hence the cost functions are:
- $$TC(q_i) = (w + r)q_i$$
- The (inverse) demand curve: $P = 1 - Q$
- Firm 1 (leader) chooses q_1 first and then firm chooses q_2

Stackelberg model - solution

- We solve the game backwards. In stage 2, firm 2 maximizes its profit:

$$\Pi_2 = (1 - q_1 - q_2)q_2 - (w + r)q_2$$

- F.O.C: $1 - 2q_2 - q_1 - (w + r) = 0$
- Best response: $q_2^R = \dfrac{1 - q_1 - (w + r)}{2}$
- Firm 1 maximizes:

$$\Pi_1 = (1 - q_1 - q_2^R - (w + r))q_1 =$$

$$= \left(1 - q_1 - \frac{1 - q_1 - (w + r)}{2} - (w + r)\right)q_1 = \frac{1 - q_1 - (w + r)}{2} q_1$$

Stackelberg-solution

- F.O.C. for firm 1

$$\frac{1-2q_1-(w+r)}{2}=0$$

- Solution:

$$q_1^S=\frac{1-(w+r)}{2}$$

$$q_2^S=\frac{1-(w+r)}{4}$$

- Firm 1 has an advantage and makes 2x more profit than firm 2

Spence-Dixit model of entry deterrence

- Step 1. An incumbent firm (1) chooses the capacity level k. Installing capacity costs r per unit.
- Step 2. A potential entrant firm (2) decides whether to enter the market or not. If enters, pays the fixed cost F.
- Step 3. Firm 1's marginal cost is w for the first k units, and $(w + r)$ for all units above k. If firm 2 stays out, firm 1 acts as a (static) monopolist. If firm 2 enters, they compete as in Cournot model, but firm 2's marginal cost is $(w + r)$ for all units.

Spence-Dixit - solution

- In step 3, if firm 2 enters, the F.O.C.s are

$$1 - 2q_1 - q_2 - w = 0$$

$$1 - 2q_2 - q_1 - w - r = 0$$

- and the equilibrium quantities are

$$q_1^D = \frac{1 - w + r}{3} \qquad q_2^D = \frac{1 - w - 2r}{3}$$

- provided that $q_1 \leq k$
- we will not worry about the case $q_1 > k$ for reasons that will become apparent

Spence-Dixit - solution

- In step 3, if firm 2 stayed out, firm 1 is a monopolist and chooses $q_1^* = \frac{1-c}{2}$ where *c* is the marginal cost
- In step 2, firm 2 enters iff $\Pi_2(q_2^D)>0$
- In step 1, firm 1 chooses *k.* There are 3 cases:
  - Blockaded entry: firm 2 will not enter even if firm 1 installs $k = 0$
  - Entry deterred: firm 1 discourages firm 2 from entering by overinvesting, i.e. choosing some $k > q_1^M$ (what a pure monopolist would produce)
  - Entry accommodated: firm 1 chooses $k = q_1^D$ and firm 1 enters

Rubinstein bargaining model

- This is a (potentially) infinitely repeated version of the ultimatum game
- Player 1 begins by offering a split of 1$ to player 2
- Player 2 accepts or rejects
- If rejects, he makes the next offer of split, except the 1$ decreases to $\$\delta$ - discount factor
- Players alternate their offers until there is an agreement

Rubinstein model - solution

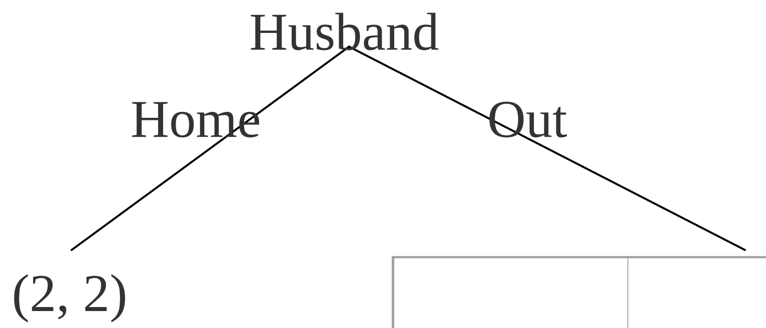
- Let $(s_t, 1\text{-}s_t)$ denote the split offered in period *t*
- Suppose that the players know that if they don't come to an agreement after 2 stages (2 offers), they will receive the split (s, 1-s)
- In stage 2,
  - player 2 is choosing between proposing an acceptable offer or getting $\delta(1\text{-}s)$ after rejection
  - the best acceptable offer is $s_2 = \delta s$ (what player 1 gets in stage 3 afetr rejection)
  - hence the offer in stage 2 will be $(\delta s, 1\text{-}\ \delta s)$
- In stage 1,
  - player 1 is choosing between proposing an acceptable offer or getting $\delta^2 s$ after rejection
  - the best acceptable offer is $s_1 = 1 - \delta(1\text{-}\ \delta s)$ (1 - what player 2 gets in stage 3 after rejection)
  - hence the offer in stage 1 will be $(1\text{-}\delta(1\text{-}\ \delta s), \delta(1\text{-}\ \delta s))$

Rubinstein - solution

- OK, but there is no final period. How do we know that *s* exists? What is it?
- Let s_H be the **highest** share that player 1 can expect in this game. By the above argument we know that the highest first-period share is $1 - \delta(1- \delta s_H)$. But since all subgames starting at odd periods look the same,
  $s_H = 1 - \delta(1- \delta s_H) \rightarrow s_H = 1/(1+ \delta)$
- Let s_L be the **lowest** share that player 1 can expect in this game. By the above argument we know that the highest first-period share is $1 - \delta(1- \delta s_L)$. But since all subgames starting at odd periods look the same,
  $s_L = 1 - \delta(1- \delta s_L) \rightarrow s_L = s_H = 1/(1+ \delta)$
- Hence the only equilibrium is for player 1 to offer $(1/(1+ \delta), \delta/(1+ \delta))$ and for player 2 to accept

Extensive-form games with imperfect information

- This game can be represented as...



		Wife	
		Football	Ballet
Hus band	Football	3, 1	0, 0
	Ballet	0, 0	1, 3

Extensive-form games with imperfect information

- This, the dotted line connects decision nodes that are in the same **information set**